

*Self-organisation, Local Symbiosis of Firms and the Life
Cycle of Localised Industrial Clusters*

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ABSTRACT. In recent years, the phenomenon of local systems with a concentration of economic activity in one or a few industries has been widely discussed in the economic literature. Several concepts have been developed to understand the success of and changes to these local systems. The various approaches focus on different local aspects and it is often claimed that the mechanisms that are responsible for the existing clusters vary too much to be treated by one approach. This paper develops a mathematical model that, on a more abstract level, captures the fundamental dynamic structure of all the observed mechanisms. On the basis of this model, the implications for the study of the evolution of such local systems is discussed.

KEYWORDS: industrial clusters, firm locality, self-organisation, evolution.
JEL classification: C60, D21, L11, R12

* I want to thank Ulrich Witt, Dirk Fornahl, Guido Büstorf and Dominic Power for their helpful comments and discussions and the German Federal Ministry for Education and Research for financial support. The usual disclaimer applies.

1. Introduction

In the recent economic as well as in the geographical literature, the phenomenon of localised industrial clusters has attracted much attention. Under the headings of ‘industrial districts’, ‘industrial clusters’, ‘innovative milieu’ and ‘regional innovative systems’, the reasons why certain regions are successful while others are not have been extensively studied. Two kinds of approaches dominate this literature: case studies of regions that are identified as being economically successful and more general theoretical approaches that aim to identify some of the circumstances that cause regions to be successful (such approaches can, for example, be found in Becattini 1990, Maillat & Lecoq 1992, Pyke & Sengenberger 1992, Scott 1992, Camagni 1995, van Dijk 1995, Markusen 1996, Lawson 1997, and Rabellotti 1997).

While the case studies often also analyse the specific circumstances that have given rise to the evolution of the studied local systems, the theoretical concepts mainly focus on the local mechanisms that make such systems successful. A general dynamic approach that explains the evolution of these systems is missing. The various local mechanisms that have been identified as playing a role within such systems have even caused some economists to argue that a general model is not feasible. This paper aims to show that such a general approach to the evolution of localised industrial clusters is feasible on an abstract level and that some fundamental insights can be deduced from it.

Models that aim to explain the phenomenon of geographic concentration on a general level, using a similar approach to that proposed here, can be found in the so-called New Economic Geography. Such models are able to explain the concentration of economic activity in certain location (see e.g. the works of Krugman 1991, Allen 1997a, Allen 1997b and Schweitzer 1998). However, although these approaches are based on similar mechanisms to the ones that are discussed in the context of localised industrial clusters, their aim is different. In these studies, a space is defined in which economic activity might increase or decrease in each location. It is shown, with the help of simulations, that local mechanisms create larger and smaller spots of economic activity with rather economically inactive places between these spots. Such approaches are well suited to explain the existence of cities, their spatial distribution and their sizes.

In the context of localised industrial clusters, however, the task is to explain, why the firms of an industry concentrate in one or a few regions. These regions are not necessarily characterised by very large cities (as can be easily seen in the cases of most Italian industrial districts) and are not distributed in some structured way

over space. If localised industrial clusters are studied, the distribution of economic activity over space is not the major point of interest. Instead, the focus has to be on one region in competition with other regions. When and how an industrial cluster evolves in one region, while all other regions remain rather inactive with respect to this industry has to be examined.

In correspondence with the numerous studies on localised industrial clusters, it is assumed here that the reasons for the evolution of such local systems are to be found mainly within the region. In addition, some global dynamics, such as changes in the global market or technological change, play a role. The spatial distribution of economic activity outside the region plays a minor role. A huge number of local mechanisms that might be responsible for the existence of localised industrial clusters are discussed in the literature (see Brenner 2000 for a discussion of the empirical evidence for the most important ones). The basic structure of these mechanisms will be identified and modelled here. The resulting model is used to deduce some fundamental insights about the evolution of such clusters.

The paper proceeds as follows. In the next section, the concept of this approach is developed. Section 3 contains the mathematical modelling of the respective processes. In Section 4, the basic characteristics of this model are examined. The life cycle of a localised industrial cluster is analysed in Section 5. The consequences of the model for the study of when, where and how localised industrial clusters evolve are discussed in Section 6. Section 7 concludes.

2. Abstract concept for local development

The numerous case studies of localised industrial clusters (see e.g. Porter 1990, Rosegrant & Lampe 1992, Saxenian 1994, Dalum 1995, Becattini 1997) reveal two basic types of mechanisms that cause the evolution of these local systems: external changes, such as the occurrence of new markets, changes in the respective markets and political measures launched by the state government, and local interactions, such as the accumulation of human capital, local information flows between firms and co-operation. In this section the task is to reduce these mechanisms to their abstract dynamics.

Before these mechanisms can be formulated in an abstract form, a variable has to be defined that reflects the influence of these mechanisms on the local economic situation. This variable should describe the state of the region in an abstract, but adequate way. Since this approach aims to explain the evolution of localised industrial clusters, the variable should allow for the detection of such clusters. Localised

industrial clusters have been defined as local systems in which the economic activity that is related to one or a few industries has increased significantly due to endogenous processes (see Brenner 2000 for a detailed discussion of this definition). The economic activity in a region might be measured by the number of employees in the industry under consideration. However, some of the mechanisms that are identified in Brenner 2000 depend on the number of firms in the region. Hence, the number of employees and the number of firms plays a role. A realistic model should contain both values. On an abstract level, as taken here, both values are merged. Their influence depends on the strength of the different local mechanisms and therefore on the industry that is considered. In a general approach, the merged variable f cannot be defined exactly. It is a function of the number of firms and the number of employees with a functional form that depends on the industry. Below we call this variable the firm population in the region and industry being considered. This variable changes if the firms in the region change in size or if firms exit or enter. All mechanisms that lead to the entry or growth of firms are modelled as increasing the local variable f . All mechanisms that cause exit or decline of firms are modelled as decreasing f .

Each of the mechanisms that influence the variable f are discussed separately below. Of course, they all work simultaneously and possibly even interactively. However, to formulate an abstract model it is helpful to separate them and model each mechanism on its own.

The analysis begins with a consideration of external changes. If, for example, the global demand for the goods that are produced by the firms in the region increases, the firms could be able to grow and/or start-ups could occur. Thus, an increase of the global market or the development of a new market will, on average, increase f . Similarly, introducing political support to the region, as in the case of Silicon Valley in the form of a huge amount of governmental contracts, leads, in general, to an increase in f . The opposite holds if political support is terminated, demand decreases or global competition becomes stronger.

However, if, for example, the demand for the produced goods increases, f will increase, but this increase will not be unlimited. Instead, abstracting from all other effects and circumstances, it can be expected that f converges to a certain value that corresponds to the external situation. Let us denote the external situation by the variable e and the corresponding state of the region, abstracting from all other processes that might interfere, by $\hat{f}(e)$. $\hat{f}(e)$ can be interpreted as the carrying capacity of a region. It depends on the external conditions, as indicated by the

dependence on e , but it also depends on regional factors. Since the regional aspects that change in the short and medium run are modelled in a separate variable below, the carrying capacity $\hat{f}(e)$ relates to all regional aspects that are constant or only change in the long run. For example, natural resources, the location of the region and the number of people within the region are such aspects. These aspects cause the carrying capacity to be different for different regions, even if the external conditions e are the same.

The local mechanisms have a slightly more complex structure. In principle, they are based on the mutual profit of firms from their co-location. However, this mutual profit can take three different forms. First, in the case of co-operation between firms, local information flows, joint use of facilities or the provision of venture capital by established firms, the firms profit directly from the proximity of other firms of the same industry. Second, the accumulation of human capital in the region, the development of an industry-specific infrastructure and public opinion are all influenced by the firms in the region and at the same time influence their opportunities to grow and the chances for new firms to enter. Finally, there are often different types of firms or different industries that rely on each other. Examples are the interactions between suppliers and buyers and the relation between certain services and their users. These three kinds of local mechanisms are discussed separately below. In the mathematical model they will be merged.

First, the size of the firm population in the region has a direct effect on the further dynamics of the firm population itself. This means that the state f has a direct effect on the changes in f . For example: the more firms that are located in the region, the more potential sources for venture capital exist which, in turn, cause more start-ups to occur, so that the number of firms increases.

Second, the size of the firm population in the region influences the development of specific conditions in this region, such as industry-specific human capital, infrastructure or public support. These local conditions, in turn, have an impact on the firm population. A second local variable is necessary to capture this aspect. This variable is denoted by c here and describes, in an abstract way, all local conditions that have an influence on the firm population. The higher c is, the more support there is for f to increase, while at the same time the value of f determines the dynamics of c .

Third, there might be other kinds of firms that profit from the particular firm population being considered and vice versa. Examples are firms that provide specific services and suppliers. If the firm population of the industry is large in a region

it enhances the establishment of related service firms and often also supplier firms in this region. Through this, two or more populations of firms become intertwined. To keep the model simple, two populations of firms that are intertwined are considered here. The size of the second firm population is denoted by s . Two situations have to be distinguished with respect to the firm population s . First, the firm population s might only provide goods or services for firms of the population f . In this case the size of s is mainly determined by the size of f (a depiction of this situation is given in Figure 1). Second, the firm population s might supply a global market that is independent of the firm population f . In this case the firm population f constitutes an additional market for their products, which is favourable due to the proximity of f (a depiction of this situation is given in Figure 2).

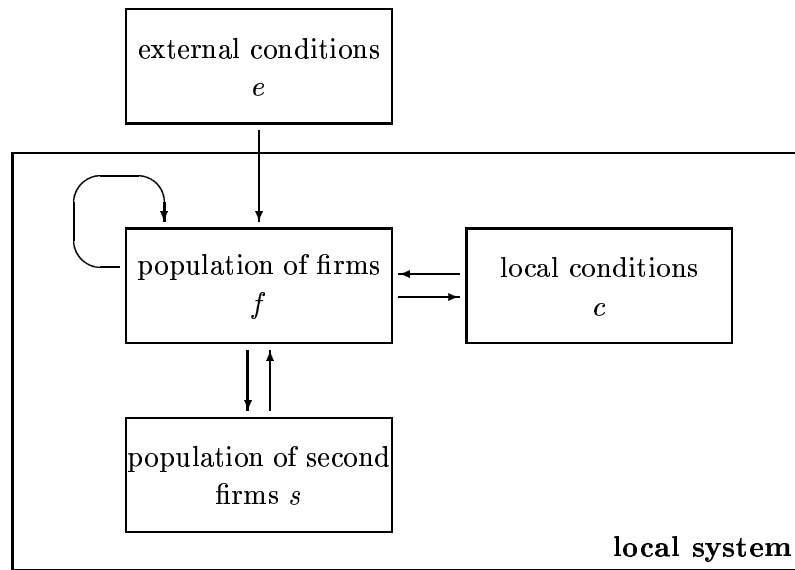


Figure 1: Structure of the interactions between the variables of the first model.

Below, the former situation will be modelled first. Once this model is analysed, the latter situation with two independent but mutually benefiting firm populations is studied.

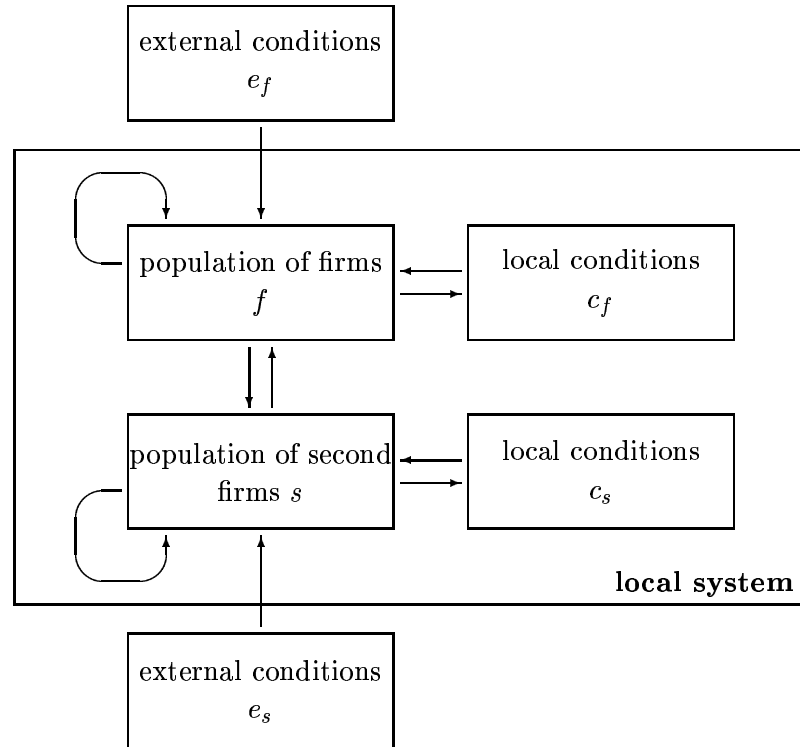


Figure 2: Structure of the interactions between the variables of the second model.

3. Mathematical model

First, the former of the two models that are described above will be analysed. This contains four variables: $f \in \mathbb{R}_+$, $e \in \mathbb{R}_+$, $s \in \mathbb{R}_+$ and $c \in \mathbb{R}_+$. However, these four variables are not treated identically. The speed of their dynamics differs. It can be assumed that the external conditions, such as market conditions, are mostly of the time quite stable compared to the population of firms, which changes continually due to exit and entry, and the local conditions. Only in the case of the emergence of new markets or significant technological changes does this hierarchy of dynamics change. Therefore, e is assumed to be a parameter in the first analysis. Later the impact of quickly changing external conditions is discussed.

The structural relations between these variables are described above. A mathematical formulation has to be found below. Figure 1 reveals that the interaction between the variables f and c and between the variables f and s can be characterised as being mutually supportive. In biology, such a process is well known under the label of symbiosis, which describes the interaction between two species that rely

on the coexistence of each other and which mutually benefit from populating the same habitat.

In theoretical biology a mathematical model has been developed to analyse the consequences of symbiotic relations. This model is given by

$$\frac{dN_1}{dt} = r_1N_1 + \alpha_1N_1N_2 - l_1N_1^2 \quad \text{and} \quad \frac{dN_2}{dt} = r_2N_2 + \alpha_2N_1N_2 - l_2N_2^2 \quad (3.1)$$

in the case of two populations with a symbiotic relationship (see Murray 1993, p. 83-84). The first terms on the right-hand sides of these equations represent the mechanism of reproduction. The second terms reflect the symbiosis between the two species. The last terms on the right-hand sides of the equations prevent the populations from growing without any limit. They reflect the limitation of the habitat.

This model can be used as a starting point for the modelling of the local symbiosis among firms and between firms and the local conditions. However, a direct transformation of the model is not adequate. Symbiotic interactions do not have to work in the same way in biology as in economics. Thus, the model (3.1) might be used to help focus on the relevant aspects, but should not simply be applied to the economic context.

There are two fundamental differences between the situation that is analysed here and the biological symbiosis that is described in Equation (3.1). First, three variables interact in a symbiotic way here: f , c and s . Second, the symbiotic interaction is not symmetric. The interaction between the firm populations f and s becomes symmetric when an independent population, s , supplying a global market, is assumed at the end of the next section. For the moment, the dynamics of the three variables are characterised by different aspects. The relevance and modelling of the three aspects that have been identified in the biological model (3.1) will be discussed for each of them below.

The effect that a large population of firms f increases the likelihood of further firms being established in the same location is mathematically similar to the biological reproduction. Thus, in Figure 1, the arrow that leads from the firm population f back to itself can be modelled by a term similar to the first one on the right-hand sides of Equations (3.1). However, there is no reason for this term to be linear in the economic case. Furthermore, such a term only appears in the case of f . There is no reason why an improvement of local conditions should automatically cause a further improvement in those conditions. In the case of the firm population s ,

it seems plausible that due to the dependence on the customers f , the population does not grow without a respective increase in f .

The second term in the biological model represents the symbiotic relationship. Its effect is, according to Equation (3.1), assumed to depend on the size of both populations linearly. This assumption does not seem to be adequate for the system that is to be studied here. It is not plausible that existing firms should benefit more from the same amount of supportive local conditions if more new firms are around. The opposite seems to be more plausible. However, for simplicity, it is assumed that the effect of local conditions on the population of firms does not depend on the state of the latter. The same holds for the benefit of firms in population f from the firms in population s and vice versa and the impact of the firm population f on the local conditions. In all four cases the impact is modelled by a term of the form $a_{cf} \cdot c(t)^{\alpha_{cf}}$. a_{cf} and α_{cf} and the respective values for the other impacts are parameters that determine the strength and functional form of the impacts.

The last term on the right-hand side of each Equation (3.1) reflects the limitations of the habitat in which the populations live. The amounts of f , c and s are limited in a similar way. The number of firms and employees in a region cannot increase without limit due to the given amount of space and the quite inflexible number of people in the region. Similarly, the improvement of the local conditions, such as infrastructure and the local availability of human capital, reaches a point where a further improvement is not feasible. Therefore, this aspect is adopted from the biological modelling of symbiosis. However, the question might be raised of whether a quadratic term is adequate here. The higher the power of this term, the later and the more abruptly the limitation interferes with a growth process. A more abrupt limitation than the one that is characterised by a quadratic function might be more adequate. Therefore, terms of the form $\phi_f \cdot f(t)^{\rho_f}$ are used below.

In addition to the interaction between the variables f , c and s , the influence of the external conditions e has to be modelled. It has been stated above that for each state e , a respective state $\hat{f}(e)$ exists. The state of the firm population is driven towards carrying capacity $\hat{f}(e)$. This effect can be included in the model by adding the term $b \cdot (\hat{f}(e) - f)$ to the equation that describes the changes in f . Thus, the following equations are obtained for the dynamics of f , c and s :

$$\frac{df(t)}{dt} = a_{ff} \cdot f(t)^{\alpha_{ff}} + a_{ef} \cdot (\hat{f}(e) - f(t)) + a_{cf} \cdot c(t)^{\alpha_{cf}} + a_{sf} \cdot s(t)^{\alpha_{sf}} - \phi_f \cdot f(t)^{\rho_f}, \quad (3.2)$$

$$\frac{dc(t)}{dt} = a_{fc} \cdot f(t)^{\alpha_{fc}} - \phi_c \cdot c(t)^{\rho_c} \quad (3.3)$$

and

$$\frac{ds(t)}{dt} = a_{fs} \cdot f(t)^{\alpha_{fs}} - \phi_s \cdot s(t)^{\rho_s} . \quad (3.4)$$

Equations (3.2), (3.3) and (3.4) describe the regional dynamics on an abstract level and will be analysed mathematically in the next section. Before this is done, a general statement can be made about the parameters that proves to be very helpful in the analysis. The parameters α_{ff} , α_{cf} , α_{sf} , α_{fc} and α_{fs} determine how high the values of f , c and s have to become before the symbiotic interactions become active. The parameters ρ_f , ρ_c and ρ_s determine how high the values of f , c and s have to become before their limitation can be felt. If the symbiotic interactions matter, they have to be relevant before the limitations interfere. Therefore, $\alpha_{ff}, \alpha_{cf}, \alpha_{sf}, \alpha_{fc}, \alpha_{fs} < \rho_f, \rho_c, \rho_s$ is assumed below.

4. Mathematical analysis of regional dynamics

The aim of this mathematical analysis is to deduce some general statements about the dynamics of local systems that are influenced by the mechanisms that are discussed above. The first step of such an analysis is the identification of the basic characteristics of the respective dynamics.

LEMMA 1: Let $\rho_f > \alpha_{ff}$, $\rho_f \cdot \rho_c > \alpha_{fc} \cdot \alpha_{cf}$ and $\rho_f \cdot \rho_s > \alpha_{fs} \cdot \alpha_{sf}$. Then, the local system, described by Equations (3.2), (3.3) and (3.4,) has either one or three stationary states within the range given by $f(t) > 0$, $c(t) > 0$ and $s(t) > 0$. In the former case this stationary state is stable, while in the latter case two of the three stationary states are stable with the unstable one located between the other two states with respect to each variable. The system never leaves the range $f(t) > 0$, $c(t) > 0$ and $s(t) > 0$ and therefore converges to one of the stable states.

The proof of Lemma 1 can be found in the appendix.

According to Lemma 1, the local system shows self-organising characteristics. Due to changes in the parameters or the exogenous condition, it might switch between two structural states: one that is characterised by only one stable stationary state and one that is characterised by two stable states. These two structural states deserve further discussion.

If there is only one stable state, the local system converges into this state. Although the modelling that is proposed here focuses on the developments in one

region, it can also be assumed to describe the dynamics of any other local system. If the regions are assumed to vary only to a small extent and the parameters are such that for all of them only one stable state exists, then all local systems converge into this unique stable state. There might be slight differences in the values of f , c and s that characterise the stable state in each of the regions. Nevertheless, the resulting global situation is characterised by a more or less uniform distribution of firms in the industry being considered. No localised industrial clusters occur in this case.

If, instead, the dynamics of a local system are characterised by two stable states, each local system might converge into a state that is characterised by a large population of firms or into a state that is characterised by a small population of firms. Again it might be assumed that all regions are similar with respect to their parameters. Then, each region might be populated by either a large number of firms or a small number. If there is only a small number of local systems that converge to the stable state with a large population of firms, these local systems can be called localised industrial clusters.

Furthermore, there might occur a situation in which two stable states exist for some local systems while only one stable state exists for others. Therefore, it is important to understand under which circumstances a local system switches from a situation with one stable state to a situation with two stable states or vice versa. This means it is necessary to determine the parameter sets for which the local system has two stable states and the parameter sets for which it has one stable state. The parameters can be classified into two groups. The parameters α_{ff} , α_{cf} , α_{sf} , α_{fc} , α_{fs} , ρ_f , ρ_c and ρ_s describe the structural characteristics of the mechanisms. a_{ff} , a_{ef} , a_{cf} , a_{sf} , a_{fc} , a_{fs} , ϕ_f , ϕ_c and ϕ_s describe their strengths. It seems adequate to assume that the structure of the mechanisms is much more stable than their strength. Therefore, a change of the parameters α_{ff} , α_{cf} , α_{sf} , α_{fc} , α_{fs} , ρ_f , ρ_c and ρ_s is excluded here. The first question to be answered is the one of whether any values of these parameters exclude the existence of two stable states or the existence of one unique stable state.

LEMMA 2: *Let $\rho_f > \alpha_{ff}$, $\rho_f \cdot \rho_c > \alpha_{fc} \cdot \alpha_{cf}$ and $\rho_f \cdot \rho_s > \alpha_{fs} \cdot \alpha_{sf}$. Then, if $\alpha_{ff} \leq 1$, $\frac{\alpha_{fc} \cdot \alpha_{cf}}{\rho_c} \leq 1$ and $\frac{\alpha_{fs} \cdot \alpha_{sf}}{\rho_s} \leq 1$, there will never exist a local system with two stable states.*

The proof of Lemma 2 is given in the appendix.

According to Lemma 2, one of the three symbiotic processes, either the one between firms within the population or the one between the population of firms and the local conditions or the one between firms of the two populations $f(t)$ and $s(t)$, has to have a power larger than one. A higher power means that the impact of the respective mechanism is comparably small as long as the conditions for this mechanism are less favourable but becomes comparably large if the mechanism becomes more active. An example for such a mechanism is the symbiotic relation between the population of firms and local public opinion. People tend not to recognise industries in which only a small number of people are employed. Thus, if the population of firms in an industry is small, these firms are barely recognised by local people. Even if the population of these firms increases, recognition does not change much as long as the population is below a certain level. However, once the population has exceeded a certain level, recognition increases rapidly. This causes increased pressure on politicians to support the respective industry, a higher willingness of the people to invest in industry-specific skills, and probably also a higher number of start-ups in this industry. Hence, this mechanism is quite sluggish in its reaction to small increases in $f(t)$ but reacts relatively more strongly to large increases in $f(t)$. According to Lemma 2, such a symbiotic mechanism is necessary for the local system to have two stable states. Symbiotic interactions that do not have such a structure do not support the existence of localised industrial clusters.

To sum up, the first condition for the existence of localised industrial clusters is that a local symbiotic interaction exists that is more sluggish in its effect at the beginning but stronger for higher values of $f(t)$ than a linear effect. It can be provided either by the symbiotic interactions within the firm population, by the interactions between the firms and the local conditions or the interactions between firms of the populations $f(t)$ and $s(t)$.

LEMMA 3: Let us call the number, stability, and location of the stationary states of the dynamics given by Equations (3.4) and (3.3) the structure of the local system. Then, the structure of the local system is influenced by the symbiotic interaction within the firm population, by the symbiotic interaction between firms and the local conditions and by the symbiotic interaction between firms of the populations $f(t)$ and $s(t)$ in an identical way. This means that for each combination of a_{ff} and α_{ff} values for the parameters a_{cf} , a_{fc} , α_{cf} and α_{fc} and for the parameters a_{sf} , a_{fs} , α_{sf} and α_{fs} , respectively, can be found

such that all three mechanism have exactly the same implications for the structure of the local system.

The proof of Lemma 3 can be found in the appendix.

Lemma 3 implies that the modelling might be restricted to the inclusion of only one of the three symbiotic mechanisms without neglecting any possible structural characteristic of the system. Thus, the local conditions $c(t)$ and the firm population $s(t)$ will be ignored mathematically in the following analysis. All statements about the symbiotic interaction within the firm population $f(t)$ that are deduced from this analysis can be transferred to the symbiotic interaction between firms and the local conditions or to the symbiotic interaction between the two firm populations $f(t)$ and $s(t)$. The dynamics are given by Equation (3.2) without the terms depending on $c(t)$ and $s(t)$.

According to Lemma 2, $\alpha_{ff} > 1$ is a necessary condition for the existence of two stable states. Given the assumption $\rho_f > \alpha_{ff}$, we obtain $\rho_f > \alpha_{ff} > 1$. For the structural characteristics of the dynamics, the exact values of these parameters are not important. Therefore, these parameters are arbitrarily set to $\alpha_{ff} = 2$, and $\rho_f = \infty$ (ϕ_f is replaced by a new parameter ϕ that is given by $\phi = \frac{1}{\phi_f}$). This simplifies the mathematical analysis without restricting the possible structural characteristics of the system. For other specifications the mathematical representation of the system can only be studied numerically. Such studies show that the system always has the same structural characteristics. Therefore, the dynamics that are given by

$$\frac{df(t)}{dt} = a_{ff} \cdot f(t)^2 + a_{ef} \cdot (\hat{f}(e) - f(t)) - \left(\frac{f(t)}{\phi}\right)^\infty \tag{4.1}$$

are studied below. With the help of this equation, the influence of the parameters a_{ff} , a_{ef} and ϕ and the exogenous conditions e can be examined.

LEMMA 4: *Let the dynamics of the local system be given by Equation (4.1) and let e be constant for a sufficiently long period of time.*

Then, the local system has two stable states if $\frac{a_{ef} + \sqrt{a_{ef}^2 - 4a_{ff}a_{ef}\hat{f}(e)}}{2\phi} < a_{ff} < \frac{a_{ef}}{4\hat{f}(e)}$. It has one stable state otherwise. In the case of two stable states, these are given by

$$\check{f}_1 = \frac{a_{ef} - \sqrt{a_{ef}^2 - 4a_{ff}a_{ef}\hat{f}(e)}}{2a_{ff}} \quad \text{and} \quad \check{f}_2 = \phi. \tag{4.2}$$

\check{f}_1 is the only stable state if and only if

$$\frac{a_{ef} - \sqrt{a_{ef}^2 - 4a_{ff}a_{ef}\hat{f}(e)}}{2\phi} < a_{ff} < \frac{a_{ef} + \sqrt{a_{ef}^2 - 4a_{ff}a_{ef}\hat{f}(e)}}{2\phi} \quad (4.3)$$

and

$$a_{ff} < \frac{a_{ef}}{4 \cdot \hat{f}(e)}, \quad (4.4)$$

while \check{f}_2 is the only stable state if and only if

$$a_{ff} < \frac{a_{ef} - \sqrt{a_{ef}^2 - 4a_{ff}a_{ef}\hat{f}(e)}}{2\phi} \quad \text{or} \quad a_{ff} > \frac{a_{ef}}{4 \cdot \hat{f}(e)}. \quad (4.5)$$

The proof of Lemma 4 is given in the appendix.

The condition for the existence of two stable states can be interpreted as follows. First, the symbiotic interactions have to be sufficiently strong. Otherwise, they are not able to overcome the tendency of the local system to adapt to the external conditions. In this case the local system converges to a stable state with a lower level of economic activity. Second, the symbiotic interactions should not be too strong compared to forces that cause the firm population to adapt to the external conditions; $a < \frac{b}{4 \cdot e(t)}$. Otherwise, the symbiotic interactions are always strong enough to increase the local economic activity up to its natural limit. In the case of Equation (4.1) the natural limit is given by ϕ . This is due to the specific choice of $\rho_f = \infty$. In general, the upper stable state is not given by a fixed value, but depends on all parameters. Hence, the fact that the upper stable state is a horizontal line in Figures 3 and 4 is a consequence of the specific choice of ρ_f .

An example of the stable and unstable stationary states dependent on the value of a_{ff} is given in Figure 3. However, the strength, a_{ff} , of local symbiotic interactions remains, in general, constant in the medium run. It might be changed by political measures (see Brenner 2001 for a detailed discussion), but in the case of spontaneously evolving localised industrial clusters, the potential for symbiotic interaction is usually given in all locations and for different times.

The exogenous conditions, e , are, in general, much more volatile. The demand for the products of the industry and the degree of competition within the industry might change quickly and to a much larger extent. Therefore, the dependence of the stable states on e will also be examined. The conditions for the existence of two stable states can be transformed into $a_{ef}\phi - a_{ff}\phi^2 < \hat{f}(e) < \frac{a_{ef}}{2a_{ff}}$. The first part of this condition is satisfied independently of e if the local symbiotic interactions,

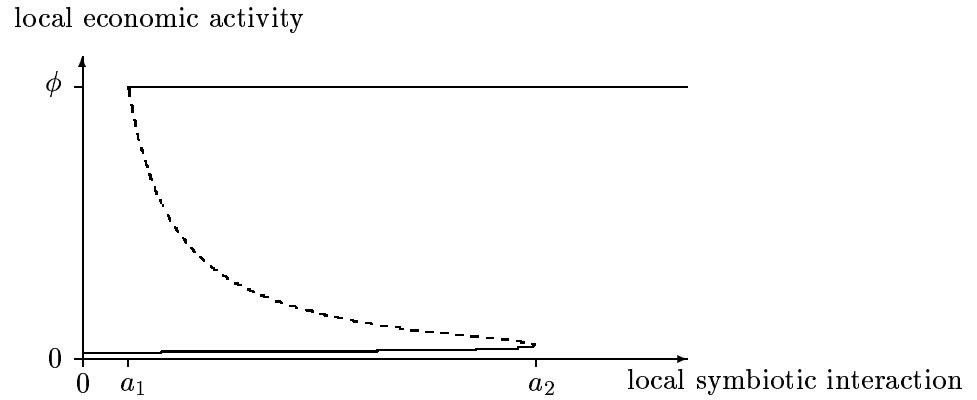


Figure 3: Stable (straight lines) and unstable (dashed line) stationary states dependent on the strength of the local symbiotic interactions.

a_{ff} , are strong enough. Otherwise a small value of $\hat{f}(e)$ leads to the existence of only one stable state which is characterised by low economic activity. If, instead, $\hat{f}(e) > \frac{a_{ef}}{2a_{ff}}$ holds, the only stable state is characterised by high economic activity. An example of the stable and unstable stationary states dependent on e is given in Figure 4.

These results imply that local economic activity shows hysteretic behaviour if e varies with time. To analyse the impact of this variation in e , we write $e(t)$ to make clear that the exogeneous conditions are now regarded to be a time-dependent variable. For small values of $e(t)$, local economic activity converges to a low level (remember that small values of e correspond to small values of $\hat{f}(e)$). An increase of $e(t)$ only slightly increases economic activity. Only if $e(t)$ passes a certain value e_2 , does the state of the local system change tremendously. It converges to the stable state with the higher level of economic activity. A later decrease of $e(t)$ decreases economic activity only gradually until another threshold e_1 is reached. If $e(t)$ decreases below this threshold, the local system switches back to the original dependence on e . This not only holds for the simplified dynamics given by Equation (4.1), but also for the general dynamics given by the Equations (3.2), (3.3) and (3.4).

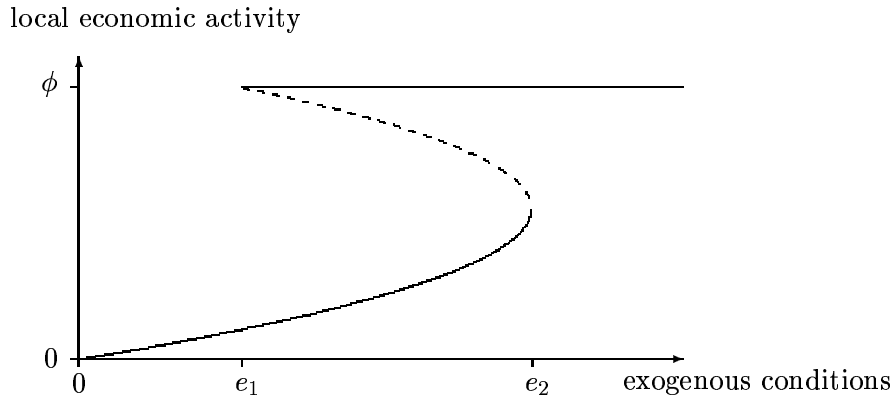


Figure 4: Stable (straight lines) and unstable (dashed line) stationary states dependent on the strength of the local symbiotic interactions.

5. Life cycle of localised industrial clusters

Above a mathematical system has been examined that is assumed to have the same structural characteristics as the processes that are involved in the evolution of localised industrial clusters. Therefore, the above findings can be used to understand the life cycle of localised industrial clusters on the basis of the hysteresis that has been found in the relationship between the exogenous conditions $e(t)$ and the stable local economic activity \check{f} . It is this hysteresis that creates the specific phenomenon that is called a localised industrial cluster. Usually it is assumed that local systems react to small changes in the external conditions by small respective changes in the state of the local system. If the changes in the external conditions are removed, the local system is expected to move back to its original state. If this common relation between two variables is not given, we talk of hysteresis. In such a case, small changes of one variable, the external conditions here, lead to small respective changes in the other variable, the state of the local system here, in most situations. However, at certain levels of the external conditions, the local system reacts to small changes with dynamics that lead it far away from the previous state. These changes in the state of the local system are sustainable in the sense that a reversion of the change does not reverse its impact on the state of the system. A huge change in the external conditions is necessary to reverse the process. This behaviour was discussed above.

According to the above study, not all dynamics that are described by the Equa-

tions (3.2), (3.3) and (3.4) show hysteresis. Hysteresis only occurs if two stable states exist for some $e(t)$. Some requirements with respect to the parameters have been identified in the above analysis. These requirements concern the parameters that determine the structure and strength of the local symbiotic interactions. In Lemma 2 it was shown that at least one of the powers α_{ff} , $\frac{\alpha_{fc} \cdot \alpha_{cf}}{\rho_c}$ and $\frac{\alpha_{fs} \cdot \alpha_{sf}}{\rho_s}$ has to be greater than 1. This means that the impact of the symbiotic interactions on the dynamics of the local system has, in comparison with a linear dependence, to be smaller for low levels of local economic activity and greater for high levels of local economic activity. Whether this requirement is satisfied depends mainly on the characteristics of the industry. How much firms can benefit from other firms at the same location and from certain local conditions and how much they induce a change in the local conditions depends little on regional aspects, but on industrial specificities, such as the necessity of skilled labour and specialised suppliers, the amount of venture capital required for start-ups, and the relevance of innovations. Hence, it can be concluded that the dynamics of local systems do not show hysteresis for all industries. Thus, localised industrial clusters might only occur in certain industries with characteristics that have to be identified in a more specific approach.

Furthermore, the strength of the local symbiotic interactions influences the occurrence of hysteresis. Although, given that the above requirements are satisfied, there is always a range of $e(t)$ for which two stable states exist, the location of this range is determined by the strength of the symbiotic interactions. If they are very weak, the values of $e(t)$ for which two stable states exist, are very high. In this case, very supportive external conditions are necessary for the local system to pass the critical level, e_2 , and transform into an industrial cluster. Therefore, the local symbiotic interactions should not be too weak if a localised industrial cluster is to evolve.

To understand the life cycle of localised industrial clusters it has to be assumed that the above requirements are satisfied for the particular industry. The initial situation is assumed to be one in which no localised cluster exists with respect to the particular industry in the specific region. It is assumed that the initial state is a stable one and that it is characterised by a low level of local economic activity in the industry concerned. There might be a high level of economic activity in the region in other industries and localised clusters of the particular industry might exist elsewhere. However, the local industry-specific economic activity is initially

low. Starting from such a situation, the evolution of a localised industrial cluster can be separated into four phases.

The first phase might be called the ‘exceeding of the critical mass’. The mathematical analysis above has shown that local economic activity converges towards the high level stable state when the external conditions exceed a certain value, e_2 , so that the low level stable state disappears, or when the state of the local system $f(t)$ is increased above the unstable stationary state. The external conditions usually become more favourable if the market for the goods of the industry expands (this includes the evolution of new markets) or if competitors decrease their supply for some reason. However, such changes have, in general, global effects. This means that other regions might profit in a similar way from these changes. New competitors and/or a growth of competitors can be expected to be the result. Therefore, a strong increase in $e(t)$ is unlikely to be sustainable. Typically the exogenous conditions $e(t)$ reach higher levels temporarily (see Process 1 in Figure 5). Such a process was, for example, observed in 1999 and 2000 in the production of mobile phones. Demand increased with such a speed that most production sites increased their output, often without being able to meet demand, as long as they produced mobile phones that were up to date. This means that the external conditions for each region were such that, given that everything outside the region remains constant, the carrying capacity of the region was very high; the value of $e(t)$ was above e_2 . As a consequence, the population $f(t)$ increases until it reaches the upper stable state (see Process 2 in Figure 5). This process can be seen as the second phase. However, after some time (in the case of mobile phones at the end of 2000) demand grows more slowly and competition becomes stronger. As a consequence, $e(t)$ falls below e_2 again. Nevertheless, in the meantime, the state of the local system has reached the temporarily unique stable state characterised by a high level of economic activity. A decrease of the exogenous conditions will not shift the local system from the higher-level stable state (see Process 3 in Figure 5), as long as $e(t)$ does not decrease below e_1 . This can be called the third phase in the life cycle of a localised industrial cluster.

The third phase is characterised by more or less stable economic activity on a high level. Changes in the external conditions and respective changes in the state of the local system might occur. As mentioned above, the line of the upper stable state does not have to be horizontal. In general, the local system will react to changes in the exogenous conditions $e(t)$, although these reactions will be small. This is due to the fact that the line of the upper stable state is never very steep

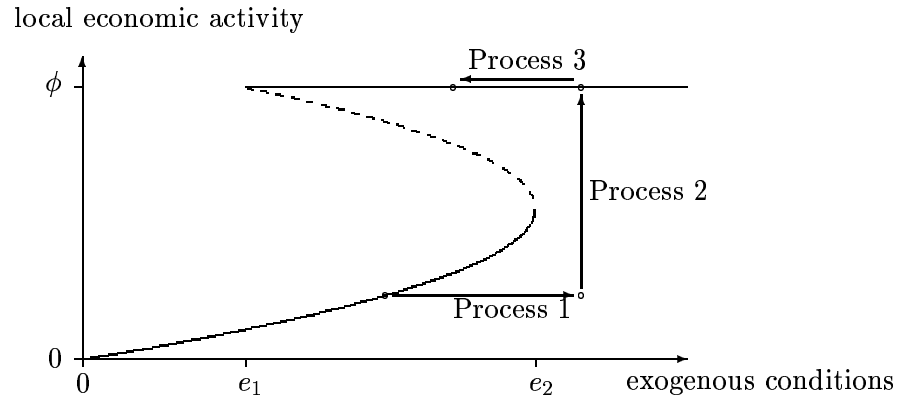


Figure 5: Stable (straight lines) and unstable (dashed line) stationary states dependent on the strength of the local symbiotic interactions and several processes that lead to the exceeding of the critical mass.

and the state of the local system moves mainly along this curve of upper stable states.

Again exogenous changes are needed to terminate the third phase. If such changes lead to a decrease of the exogenous conditions, $e(t)$, below the value e_1 , the state of the local system converges to the low-level stable state and the localised industrial cluster disappears. This is called the fourth phase. The usual reason for such a change in the exogenous conditions is the emergence of a new technology in connection with the appearance of new products that replace old ones. This makes the external conditions less favourable for firms that produce the old goods. There are two possibilities: Either these firms are able to adopt the new technology or they lose, at least, the major share of their markets. The latter implies that $e(t)$ decreases tremendously for these firm and the respective cluster. However, the local conditions $c(t)$ are still very favourable and the population of firms is large. If the firms manage to move to a different market before their decline has been too severe, the localised industrial cluster remains intact. Even if the firms are not able to prevent their decline, other firms might step in and create a new localised industrial cluster while the old one disappears. A disappearing industrial cluster always leaves behind a region with favourable local conditions that remain for some time. This is a good starting point for the evolution of a new localised industrial cluster.

6. Implications for the study of the evolution of localised industrial clusters

The aim of this approach was to develop an abstract concept that can be used as a framework for research into the evolution of localised industrial clusters. Of course, such an approach omits many details and is not able to explain the evolution of each specific localised industrial cluster in detail. Like every general theory, it is not well suited to dealing with specific developments and aspects. Nevertheless, it is able to guide the research into such specific processes. First, it allows separation of the research into the studies of different aspects. Second, it provides some fundamental knowledge about the characteristics of the mechanisms that play a role at different phases of the development. Hence, it provides some criteria for the identification of the respective mechanisms.

In the literature several questions have been posed in the context of the evolution of localised industrial clusters. The most common questions are those about when, where and how localised industrial clusters evolve, and how they can be created by political measures. The concept that has been developed above allows partial separation of these questions. It showed that the evolution of localised industrial clusters is the outcome of the co-existence of two kind of processes: the local symbiotic interactions and the exceeding of the critical mass by the exogenous conditions.

Symbiotic interactions within a region are necessary for the existence of two stable states. These endogenous dynamics are responsible for the fact that localised industrial clusters exist. They partly answer the question of when such clusters evolve. Since these dynamics depend on industrial-specific characteristics, localised clusters will not evolve in all industries. A change in the characteristics of an industry might cause the evolution of a localised cluster. Thus, to answer the question of when localised industrial clusters evolve, the industry characteristics that are necessary for their existence have to be studied.

The approach that is developed here gives some first hints for such an identification. Mechanisms that cause a symbiotic relation either within the local firm population or between the firm population and the local conditions or other local firm populations have to be identified. A symbiotic relation is given if an increase of one element causes a growth of the other element and vice versa. Thus, four kinds of causal relations have to be investigated: First, the causal relations between the size of the local firm population and its change, determined by entry, exit and growth of firms: Second, the causal relations between the size of the local firm pop-

ulation and the change in local conditions: Third, the causal relations between the state of the local conditions and the change in the local firm population: Fourth, causal relations between two populations of firms, such as the relation between manufacturing firms and the respective service firms. A further requirement for the mechanisms that cause a symbiotic relation between these elements has been identified above. The effects of the various mechanisms have to increase with an increasing slope for a larger size of the firm population or a more supportive state of the local conditions, respectively. These findings can be used to test whether the numerous mechanisms that have been claimed in the literature to be responsible for the evolution of localised industrial clusters might indeed be able to cause such an evolution.

While the endogenous dynamics are important for the question of when and how localised industrial clusters evolve, they offer no answer, or a very incomplete answer, to the question of where localised clusters evolve. There is some dependence of the symbiotic relations on location-specific characteristics. Thus, there might be regions in which local symbiotic relations might not exist to a sufficient extent. These regions can be excluded from the set of regions that are considered as possible locations for an industrial cluster. However, sufficient symbiotic relations do not cause the evolution of an industrial cluster. In addition, the local state has to exceed a critical mass due to external changes. However, the local symbiotic relations determine the value of the critical mass and therefore the likelihood of exceeding this critical mass at certain locations. This might be important information if it is intended to use political measures to create localised industrial clusters.

The question of how localised industrial clusters evolve is answered by the complete concept that is developed above. The fundamental mechanisms that are responsible for the evolution of localised industrial clusters have been outlined above, while the details of these mechanisms have to be elaborated in further studies.

The process of exceeding the critical mass determines where localised industrial clusters occur. Once the exogenous conditions exceed the critical mass, the endogenous dynamics push the local system towards the high-level stable state and stabilise this state. The exogenous conditions, in turn, are dependent on the strength of competition in the respective market. Once localised industrial clusters have occurred elsewhere, competition becomes stronger and the external conditions become less favourable. Thus, the more clusters that have already evolved, the less likely is the evolution of further clusters in this industry. The question of where localised industrial clusters evolve can thus be reformulated into the question of

where the critical mass is most likely to be exceeded first. Therefore, the processes that lead to the exceeding of the critical mass have to be understood more comprehensively to answer the question of why localised industrial clusters occur in certain places and not in others. A better understanding of these processes also helps to identify adequate political measures for the creation of localised industrial clusters. However, further examination of these processes is beyond the scope of this paper.

7. Conclusions

An abstract concept for the study of localised industrial clusters has been proposed here. It has been shown that the phenomenon of localised industrial clusters can be understood on a general level and that the life cycle of such clusters can be explained on this basis.

This concept can be used as a basis for further research into localised industrial clusters. Some of the fundamental characteristics of the processes that are involved in the evolution of such clusters have been identified. First, it has been shown that two processes can be studied separately: the mechanisms that lead to the exceeding of the critical mass and the mechanisms that are responsible for the local symbiotic interactions. For both mechanisms some necessary characteristics that will help to define these mechanisms more precisely have been identified. Thus, the concept developed here provides a valuable framework for the identification of the relevant mechanisms for specific developments in localised industrial clusters and a generalising structure that helps to see the similarities in different historical developments.

Nevertheless, it only provides a framework. To understand the evolution of localised industrial clusters in more detail, the respective mechanisms have to be studied more comprehensively. Combining the general approach that is taken here with the very specific analyses that are found in the numerous case studies is promising in this context. This has been done for the local symbiotic interactions in Brenner 2000. A similar analysis of the processes that lead to the exceeding of the critical mass has still to be done.

A. Proofs

Proof of Lemma 1:

The stationary states $(\check{f}, \check{c}, \check{s})$ of the dynamic system (3.2), (3.3) and (3.4) have to

satisfy $\left. \frac{df(t)}{dt} \right|_{st.} = 0$, $\left. \frac{dc(t)}{dt} \right|_{st.} = 0$ and $\left. \frac{ds(t)}{dt} \right|_{st.} = 0$ (*st.* stands for a stationary state given by $f(t) = \check{f}$, $c(t) = \check{c}$ and $s(t) = \check{s}$). This implies the conditions

$$\check{c} = \left(\frac{a_{fc}}{\phi_c} \right)^{\frac{1}{\rho_c}} \check{f}^{\frac{\alpha_{fc}}{\rho_c}}, \quad (\text{A.1})$$

$$\check{s} = \left(\frac{a_{fs}}{\phi_s} \right)^{\frac{1}{\rho_s}} \check{f}^{\frac{\alpha_{fs}}{\rho_s}} \quad (\text{A.2})$$

and

$$a_{ff} \cdot \check{f}^{\alpha_{ff}} + a_{ef} \cdot e - a_{ef} \cdot \check{f} + a_{cf} \cdot \check{c}^{\alpha_{cf}} + a_{sf} \cdot \check{s}^{\alpha_{sf}} - \phi_f \cdot \check{f}^{\rho_f} = 0. \quad (\text{A.3})$$

Inserting Equations (A.1) and (A.2) into Equation (A.3) results in

$$a_{ff} \cdot \check{f}^{\alpha_{ff}} + a_{ef} \cdot e - a_{ef} \cdot \check{f} + a_{cf} \left(\frac{a_{fc}}{\phi_c} \right)^{\frac{\alpha_{cf}}{\rho_c}} \check{f}^{\frac{\alpha_{cf} \cdot \alpha_{fc}}{\rho_c}} + a_{sf} \left(\frac{a_{fs}}{\phi_s} \right)^{\frac{\alpha_{sf}}{\rho_s}} \check{f}^{\frac{\alpha_{sf} \cdot \alpha_{fs}}{\rho_s}} - \phi_f \cdot \check{f}^{\rho_f} = 0. \quad (\text{A.4})$$

Equation (A.4) determines the values of \check{f} for all stationary states. For each value of \check{f} , exactly one value of \check{c} and \check{s} is defined by Equations (A.1) and (A.2). Therefore, the dynamics given by (3.2), (3.3) and (3.4) have exactly the same number of stationary states as there are solutions to Equations (A.4). Let us assume that the parameters α_{ff} , α_{cf} , α_{sf} , α_{fc} , α_{fs} , ρ_f , ρ_c and ρ_s are rational values. Then, a natural number η can be found such that $\alpha_{ff} \cdot \eta = h_f$, $\frac{\alpha_{fc} \cdot \alpha_{cf}}{\rho_c} \cdot \eta = h_c$, $\frac{\alpha_{fs} \cdot \alpha_{sf}}{\rho_s} \cdot \eta = h_s$ and $\rho_f \cdot \eta = h_\rho$ are natural numbers. A new variable x is defined by $x = \check{f}^{\frac{1}{\eta}}$. Equation (A.4) then reads

$$a_{ff} \cdot x^{h_f} + a_{ef} \cdot e - a_{ef} \cdot x^\eta + \left(\frac{a_{fc}}{\phi_c} \right)^{\frac{\alpha_{cf}}{\rho_c}} x^{h_c} + \left(\frac{a_{fs}}{\phi_s} \right)^{\frac{\alpha_{sf}}{\rho_s}} x^{h_s} - \phi_f \cdot x^{h_\rho} = 0. \quad (\text{A.5})$$

The left-hand side of Equation (A.5) is a polynomial of order h_ρ . The h_ρ -th order derivative of this polynomial is a constant function (the h_ρ -th order derivative of the last term in the polynomial) that has no zero. Let us now consider each derivative in turn, from the h_ρ -th order derivative to the first-order derivative of the polynomial and consider all zero x_0 with $x_0 > 0$. The r -th order derivative of the polynomial is denoted by $p_r(x)$ in the following. Let us assume that $p_r(x)$ has n_r zeros for $x > 0$. $p_{r-1}(x)$ has either the same number of terms or one more than $p_r(x)$. If it has the same number of terms, there is one zero at $x = 0$. In this case, the number n_{r-1} is at most as high as n_r , because $p_r(x)$ is the derivative of $p_{r-1}(x)$ which implies that $p_{r-1}(x)$ has exactly n_r extreme points for $x > 0$. If $p_{r-1}(x)$ has one term more than $p_r(x)$, this term is a constant. Hence, $p_{r-1}(x) \neq 0$. Nevertheless, $p_{r-1}(x)$ has n_r

extreme points for $x > 0$. In addition, every polynomial $p_{r-1}(x)$ becomes negative for $x \rightarrow \infty$. Therefore, the number of zeros is either $n_{r-1} = n_r$, if either n_r is even and $p_{r-1}(0) > 0$ or n_r is odd and $p_{r-1}(0) < 0$, or $n_{r-1} = n_r + 1$, if either n_r is even and $p_{r-1}(0) < 0$ or n_r is odd and $p_{r-1}(0) > 0$. There are two negative and four positive terms on the right hand side of Equation (A.5). The last term determines the polynomial $p_{h_p}(x)$ which we start with. Therefore, in one case $p_{r-1}(0) < 0$ holds and in four cases $p_{r-1}(0) > 0$ holds. As a consequence, there are at most three cases in which the number of zeros increases by one. Thus, this polynomial has at most three zeros with $x > 0$.

The fact that the dynamics given by (3.2), (3.3) and (3.4) never leave the range $f(t) > 0$, $c(t) > 0$ and $s(t) > 0$ can be easily verified: $\frac{df(t)}{dt}$ becomes positive if $f(t)$ approaches zero, while $\frac{dc(t)}{dt}$ and $\frac{ds(t)}{dt}$ become positive if $c(t)$ and $s(t)$, respectively, approach zero, as long as $s(t) > 0$. Furthermore, the conditions $\rho_f > \alpha_{ff}$, $\rho_f \cdot \rho_c > \alpha_{fc} \cdot \alpha_{cf}$ and $\rho_f \cdot \rho_s > \alpha_{fs} \cdot \alpha_{sf}$ imply that the last term on the right-hand side of Equation (3.4) dominates the dynamics for sufficiently large values of $f(t)$. As a consequence, the variables are bounded from below and above. This also implies that there has to be at least one stable state or limit cycle.

To examine the stability of the stationary states, the Jacobi-matrix is calculated. The zeros are ordered according to their value of f and numbered by 1, 2, and 3. For the first and the third zero, the Equations (3.2), (3.3) and (3.4) imply

$$\left. \frac{\frac{df(t)}{dt}}{df(t)} \right|_{st.} < 0 \quad \left. \frac{\frac{df(t)}{dt}}{dc(t)} \right|_{st.} > 0 \quad \left. \frac{\frac{df(t)}{dt}}{ds(t)} \right|_{st.} > 0, \quad (\text{A.6})$$

$$\left. \frac{\frac{dc(t)}{dt}}{df(t)} \right|_{st.} > 0 \quad \left. \frac{\frac{dc(t)}{dt}}{dc(t)} \right|_{st.} < 0 \quad \left. \frac{\frac{dc(t)}{dt}}{ds(t)} \right|_{st.} = 0 \quad (\text{A.7})$$

and

$$\left. \frac{\frac{ds(t)}{dt}}{df(t)} \right|_{st.} > 0 \quad \left. \frac{\frac{ds(t)}{dt}}{dc(t)} \right|_{st.} = 0 \quad \left. \frac{\frac{ds(t)}{dt}}{ds(t)} \right|_{st.} < 0. \quad (\text{A.8})$$

Furthermore, the following relations hold:

$$-\left. \frac{\frac{dc(t)}{dt}}{dc(t)} \right|_{st.} > \left. \frac{\frac{dc(t)}{dt}}{df(t)} \right|_{st.}, \quad -\left. \frac{\frac{ds(t)}{dt}}{ds(t)} \right|_{st.} > \left. \frac{\frac{ds(t)}{dt}}{df(t)} \right|_{st.} \quad (\text{A.9})$$

and

$$-\left. \frac{\frac{df(t)}{dt}}{df(t)} \right|_{st.} > \left. \frac{\frac{df(t)}{dt}}{dc(t)} \right|_{st.} + \left. \frac{\frac{df(t)}{dt}}{ds(t)} \right|_{st.}. \quad (\text{A.10})$$

The Conditions (A.9) and (A.10) imply

$$-\left.\frac{df(t)}{dt}\right|_{st.} > -\frac{\left.\frac{df(t)}{dc(t)}\right|_{st.} \cdot \left.\frac{dc(t)}{df(t)}\right|_{st.}}{\left.\frac{dc(t)}{dc(t)}\right|_{st.}} + \frac{\left.\frac{df(t)}{ds(t)}\right|_{st.} \cdot \left.\frac{ds(t)}{df(t)}\right|_{st.}}{\left.\frac{ds(t)}{ds(t)}\right|_{st.}}. \quad (\text{A.11})$$

To prove that all eigenvalues are negative for the first and third zero, it will now be shown that condition (A.11) contradicts the assumption of a positiv eigenvalue. Therefore, let us assume that a positiv eigenvalue $\lambda > 0$ exists. The corresponding eigenvalue is denoted by $\mathbf{v} = (1, v_2, v_3)$. The conditions (A.6), (A.7) and (A.8) imply that $v_2 > 0$ and $v_3 > 0$. Furthermore, $\mathbf{J}\mathbf{v} = \lambda\mathbf{v}$ has to hold, where \mathbf{J} denotes the Jacobi-matrix. This condition leads to

$$-v_2 \cdot \left.\frac{dc(t)}{df(t)}\right|_{st.} < \left.\frac{dc(t)}{df(t)}\right|_{st.}, \quad -v_3 \cdot \left.\frac{ds(t)}{df(t)}\right|_{st.} < \left.\frac{ds(t)}{df(t)}\right|_{st.} \quad (\text{A.12})$$

and

$$-\left.\frac{df(t)}{df(t)}\right|_{st.} < v_2 \cdot \left.\frac{df(t)}{dc(t)}\right|_{st.} + v_3 \cdot \left.\frac{df(t)}{ds(t)}\right|_{st.}. \quad (\text{A.13})$$

The Conditions (A.12) and (A.13) imply

$$-\left.\frac{df(t)}{df(t)}\right|_{st.} < -\frac{\left.\frac{df(t)}{dc(t)}\right|_{st.} \cdot \left.\frac{dc(t)}{df(t)}\right|_{st.}}{\left.\frac{dc(t)}{dc(t)}\right|_{st.}} + \frac{\left.\frac{df(t)}{ds(t)}\right|_{st.} \cdot \left.\frac{ds(t)}{df(t)}\right|_{st.}}{\left.\frac{ds(t)}{ds(t)}\right|_{st.}}, \quad (\text{A.14})$$

which contradicts Condition (A.11). Thus, all eigenvalues of the first and last stationary state are negative and these states are stable. In the case of the second stationary state the condition

$$\left.\frac{df(t)}{df(t)}\right|_{st.} < 0 \quad (\text{A.15})$$

does not necessarily hold. However, either the product of the eigenvalues or the sum of the eigenvalues is negative, so that at least one of the eigenvalues is negative. Due to the fact that both of the other stationary states are stable, at least one of the eigenvalues of the second stationary state is positive. Therefore, the second stationary state is a saddle-point. ■

Proof of Lemma 2: Let us assume that $\alpha_{ff} \leq 1$, $\frac{\alpha_{cf} \cdot \alpha_{fc}}{\rho_c} \leq 1$ and $\frac{\alpha_{sf} \cdot \alpha_{fs}}{\rho_s} \leq 1$. Furthermore, $\alpha_{ff} > 0$, $\frac{\alpha_{cf} \cdot \alpha_{fc}}{\rho_c} > 0$ and $\frac{\alpha_{sf} \cdot \alpha_{fs}}{\rho_s} > 0$ holds due to the definition of the parameters. The second derivative on the left-hand side of Equation (A.4) reads

$$\begin{aligned}
& a_{ff} \cdot \alpha_{ff} \cdot (\alpha_{ff} - 1) \cdot \check{f}^{\alpha_{ff}-2} + a_{cf} \left(\frac{a_{fc}}{\phi_c} \right)^{\frac{\alpha_{cf}}{\rho_c}} \cdot \frac{\alpha_{cf} \cdot \alpha_{fc}}{\rho_c} \cdot \left(\frac{\alpha_{cf} \cdot \alpha_{fc}}{\rho_c} - 1 \right) \cdot \check{f}^{\frac{\alpha_{cf} \cdot \alpha_{fc}}{\rho_c} - 2} \\
& + a_{sf} \left(\frac{a_{fs}}{\phi_s} \right)^{\frac{\alpha_{sf}}{\rho_s}} \cdot \frac{\alpha_{sf} \cdot \alpha_{fs}}{\rho_s} \cdot \left(\frac{\alpha_{sf} \cdot \alpha_{fs}}{\rho_s} - 1 \right) \cdot \check{f}^{\frac{\alpha_{sf} \cdot \alpha_{fs}}{\rho_s} - 2} - \phi_f \cdot \rho_f \cdot (\rho_f - 1) \cdot \check{f}^{\rho_f - 2}
\end{aligned} \tag{A.16}$$

which is negative for all values of \check{f} . Therefore, the left-hand side of Equation (A.4) has at most one zero with $\check{f} > 0$ (remember that this function is positive for $\check{f} = 0$ and negative for sufficiently high values of \check{f}). ■

Proof of Lemma 3: The location of the stationary states with respect to the variable $f(t)$ is determined by Equation (A.4). The number of stationary states equals the number of solutions for Equation (A.4) and their stability is determined by their number. Thus, these aspects are identical for the two systems whenever Equation (A.4) is identical for the two systems. It is obvious that for every a_{cf} , a_{fc} , α_{cf} , α_{fc} and ρ_c an a_{ff} and α_{ff} can be chosen such that the two terms, $a_{ff} \cdot \check{f}^{\alpha_{ff}}$ and $a_{cf} \left(\frac{a_{fc}}{\phi_c} \right)^{\frac{\alpha_{cf}}{\rho_c}} \check{f}^{\frac{\alpha_{cf} \cdot \alpha_{fc}}{\rho_c}}$, in Equation (A.4) are identical. The same holds for the parameters a_{sf} , a_{fs} , α_{sf} , α_{fs} and ρ_s and the respective term in Equation (A.4) ■

Proof of Lemma 4: The stationary states for the dynamics given by Equation (4.1) are given by

$$a_{ff} \cdot \check{f}^2 + a_{ef} \cdot (\hat{f}(e) - \check{f}) - \left(\frac{\check{f}}{\phi} \right)^\infty = 0. \tag{A.17}$$

The left-hand side of this equation is determined by the first two terms for all values of \check{f} that satisfy $0 < \check{f} < \phi$. For values of $\check{f} > \phi$, the left hand-side is negative. Therefore, solutions for Equation (A.17) can either be found in the range of $0 < \check{f} < \phi$ or at $\check{f} = \phi$ if $a_{ff} \cdot \phi^2 + a_{ef} \cdot (\hat{f}(e) - \phi) > 0$. For $0 < \check{f} < \phi$, the last term in Equation (A.17) is zero, so that a quadratic equation results that is solved by

$$\check{f} = \frac{a_{ef} \pm \sqrt{a_{ef}^2 - 4a_{ff}a_{ef}\hat{f}(e)}}{2a_{ff}}. \tag{A.18}$$

Therefore, there are three solutions if $a_{ef} > 4a_{ff}\hat{f}(e)$. However, these are only solutions to Equation (A.17) if they satisfy $0 < \check{f} < \phi$. The following situations might occur: 1) If none of the values of (A.18) satisfies $\check{f} < \phi$, $\check{f} = \phi$ is the only solution of Equation (A.17) and therefore the only stationary state. 2) If one of the values of (A.18) satisfies $\check{f} < \phi$, this is the only stationary state. 3) If two of the

values of (A.18) satisfy $\check{f} < \phi$, both are stationary states and there is an additional stationary state at $\check{f} = \phi$. 4) If three of the values of (A.18) satisfy $\check{f} < \phi$, these are also the stationary states of Equation (4.1). The stability of these states is given according to Lemma 1. ■

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